Appendix

The computation of attenuation factors is lengthy and complicated. For image lines which have dielectrics with semicircular cross sections, the contributions to attenuation due to dielectric loss (α_d) and conduction loss (α_c) may be found from the following expressions:

$$\alpha_d = 27.3\left(\frac{\Phi\epsilon}{\lambda}\right)R \quad \text{decibels/meter} \tag{1}$$

$$\alpha_c = 69.5\left(\frac{R_s R'}{\eta\lambda}\right) \quad \text{decibels/meter} \tag{2}$$

where

Φ = loss tangent of the dielectric rod
ϵ = relative dielectric constant of the rod
λ = free-space wavelength (meters)
η = intrinsic impedance of free space
R_s = surface resistivity of the image plane.

The factors R and R' are complicated functions of the dielectric constant and diameter (in free-space wavelengths) of the rod. Explicit expressions for R and R' may be found in the paper by King and Schlesinger.[2]

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Nearly all of the experimental measurements were carried out by James D. Rodgers, whose care and patience are much appreciated.

The Interaction of Microwaves with Gas-Discharge Plasmas*

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Summary—**The interaction of microwaves with gas-discharge plasmas provides a valuable tool for studying the fundamentals of gas-discharge phenomena and methods of controlling and switching microwave power. A summary of our present state of knowledge in this field is presented by using as particular examples the interaction of high density and low density gas-discharge plasmas in S-band resonant cavities, both in the presence and absence of dc magnetic fields.**

Introduction

The effective dielectric coefficient of a plasma[1] in the absence of a magnetic field is given by

$$K = 1 - \left[\frac{\omega_p^2}{\omega^2}\,\frac{1 + j\dfrac{\nu_c}{\omega}}{1 + \left(\dfrac{\nu_c}{\omega}\right)^2}\right]. \tag{1}$$

Here ω_p is the plasma frequency given by the relation $\omega_p{}^2 = ne^2/m\epsilon_0$; ω is the applied radian frequency, and ν_c is the collision frequency of electrons in the gas given by ν_c = (constant) p, where p is the pressure. The square root of the dielectric coefficient (1) is related to the attenuation and phase shift of a plane wave, as represented in Fig. 1. This figure is calculated for the specific case of hydrogen gas at a microwave frequency of 4500 mc. In the low density region the attenuation and the phase shift are linear functions of the density, but at higher densities this linearity disappears and the functional relation becomes more complicated. In the usual use of microwave techniques for the diagnostic studies of plasmas, a restriction is placed on the method by the complexities of the solution in high density regions where the linear dependence does not hold. Usually, the microwave technique is restricted to the low density region well below the plasma frequency at which $\omega_p/\omega = 1$.

The solution shown in Fig. 1 is valid in the absence of a magnetic field. If a magnetic field is applied, the dielectric coefficient depends not only upon the density and magnitude of the magnetic field, but also on the geometrical configuration that is under consideration and the direction of propagation of the electromagnetic wave with respect to the magnetic field. Four cases can be distinguished for the purpose of simplifying the discussion; they are given in the following equations.[2]

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[1] H. Margenau, "Conduction and dispersion of ionized gases at high frequencies," *Phys. Rev.*, vol. 69, pp. 508–513; May, 1946.

[2] W. P. Allis, "Motions of Ions and Electrons" in "Handbuch der Physik," Springer Verlag, Berlin, Ger., vol. 21, pp. 383–444; 1956.

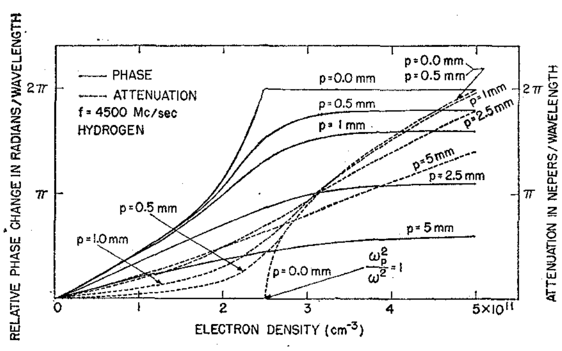

Fig. 1—Phase shift and attenuation of a plane wave as a function of electron density.

Propagation along B Field

Right-handed circularly polarized plane wave:

$$K_r = 1 - \frac{\omega_p^2}{\omega^2} \frac{1}{\left(1 - \frac{\omega_b}{\omega}\right) - j\frac{\nu_c}{\omega}}. \tag{2a}$$

Left-handed circularly polarized plane wave:

$$K_l = 1 - \frac{\omega_p^2}{\omega^2} \frac{1}{\left(1 + \frac{\omega_b}{\omega}\right) - j\frac{\nu_c}{\omega}}. \tag{2b}$$

Propagation Perpendicular to B Field

E field parallel to B:

$$K_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2} \frac{1}{1 - j\frac{\nu_c}{\omega}}. \tag{2c}$$

E field perpendicular to B:

$$K_{\perp} = \frac{2K_rK_l}{K_r + K_l} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega_p^2 - \omega^2}{\omega_p^2 + \omega_b^2 - \omega^2}. \tag{2d}$$

Here the symbol ω_b refers to the cyclotron frequency eB/m. In (2d) there is a component of E field along the B field so that the E field is not divergenceless. In this equation, also, the approximation is written in the absence of collision $\nu_c = 0$.

In a guiding structure such as a microwave cavity the dielectric coefficient must be represented by one or more of the coefficients mentioned above. As a result, it is not possible, even at low electron densities, to obtain a general theory in a form that is suitable for experimental verification and use in the microwave diagnostics of a magnetized plasma which is valid for all possible configurations of the microwaves. Consequently, only a few special configurations of the microwave field are analyzed. The discussion is restricted to a narrow cylindrical plasma column placed coaxially in a cylindrical microwave cavity. The behavior of the modes TM_{111}, TE_{011}, and TM_{010} or TM_{020} is considered. The static magnetic field, in all cases, is applied along the axis of the cavity.

TM_{111} Mode (Degenerate Modes)

Low Electron Densities

When the measuring mode is degenerate in frequency, the nonisotropic plasma removes the degeneracy. The physical reason for this can be seen when the TM_{111} mode is considered. When the radius of the plasma column is small compared with the cavity radius, the E field of the TM_{111} mode can be considered as linearly polarized in the plasma region. A linearly polarized field can, in turn, be considered as composed of two circularly polarized fields rotating in opposite directions. As mentioned earlier, the refractive index of the plasma is different for the two fields. As a result, the resonant frequency of the cavity splits into two frequencies, and the frequency shifts are given by[3]

$$\left(\frac{\Delta f}{f}\right)_{\circlearrowright} \cong \frac{\omega_p^2}{\omega^2}\left[\frac{\left(1 - \frac{\omega_b}{\omega}\right)}{\left(1 - \frac{\omega_b}{\omega}\right)^2 + \frac{\nu_c^2}{\omega^2}}\right]$$

$$\left(\frac{\Delta f}{f}\right)_{\circlearrowleft} \cong \frac{\omega_p^2}{\omega^2}\left[\frac{\left(1 + \frac{\omega_b}{\omega}\right)}{\left(1 + \frac{\omega_b}{\omega}\right)^2 + \frac{\nu_c^2}{\omega^2}}\right]. \tag{3}$$

The subscript arrows denote the rotation of the E field and the rotation of the electrons in the magnetic field. Thus in the first equation the E field is rotating in the same direction as the electrons; and in the second equation, the E field is considered as rotating in the opposite direction from the electrons. The solution for this set of equations is given graphically in Fig. 2. Also shown in this figure is a diagram of the TM_{111} mode, in which the electric field is represented by solid lines, and the plasma on the axis of the cylindrical cavity is represented by a shaded circle. The magnetic field is perpendicular to the plane of the figure. Fig. 2 gives the solution of (3) for the resonant-frequency shift as a function of electron density with ω_b/ω kept constant. If the equations for the resonant frequency are solved as a function of the ratio ω_b to ω for a constant density, the result is as shown in Fig. 3. It is obvious from this figure that for the case of the electric field rotating in the same direction as the electrons, the frequency shift is a complicated function of the magnetic field applied to the plasma.

High Electron Densities

When the electron densities are high, $\omega_p > \omega$, the relations for the frequency shift are invalid, mainly because the field in the plasma cannot be approximated at high electron density by the field in the absence of a plasma, the approximation which was made in the for-

[3] J. C. Slater, "Microwave electronics," *Revs. Mod. Phys.*, vol. 18, pp. 441–512; October, 1946.

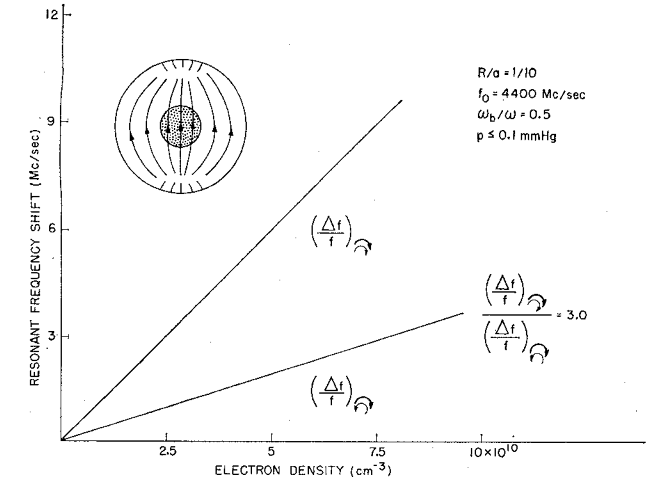

Fig. 2—Resonant frequency shift as a function of electron density for the TM_{111} mode. The magnetic field is perpendicular to the plane of the figure.

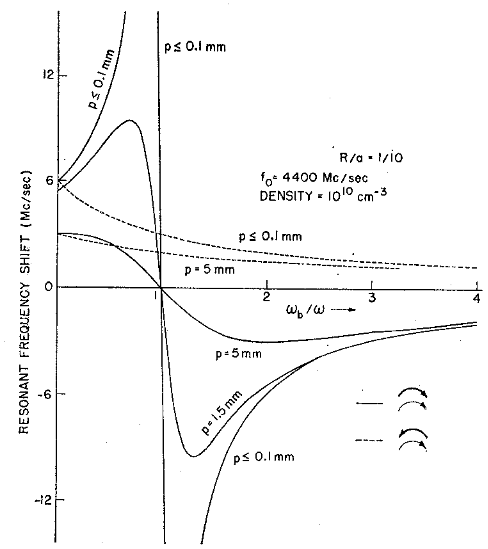

Fig. 3—Frequency shift as a function of magnetic field for the TM_{111} mode.

mer case. By far the most difficult problem is that of the TM_{111} mode. Here the field in the plasma is no longer linearly polarized but, because the skin depths for the left and right-handed rotating waves are different, the field is elliptically polarized. The degree of ellipticity is a function of the radius and of the density distribution. The exact solutions to this problem therefore, have not been attempted.

TE_{011} Mode (Nondegenerate Modes)

Low Electron Densities

In this case the perturbation theory can be extended in a straightforward manner for taking into account the presence of the static magnetic field. The dielectric coefficient becomes a tensor. At low electron densities, the field in the plasma can be approximated by the field in the absence of the plasma, so that only one diagonal component of the tensor needs to be considered. For very low pressures, the frequency shift becomes

$$\frac{\Delta f}{f} \sim \frac{\omega_p^2}{\omega^2} \frac{\omega^2[\nu_c^2 + \omega^2 - \omega_b^2]}{[\nu_c^2 + (\omega + \omega_b)^2][\nu_c^2 + (\omega - \omega_b)^2]}. \qquad (4)$$

Eq. (4) exhibits an interesting property of the TE_{011} mode, namely, that the frequency shift is zero when $\omega_b^2 = \omega^2 + \nu_c^2$ independent of the electron density. This has been used as a direct measure of ν_c.

High Electron Densities

The TE_{011} mode is the mode that is ideally suited for measurement of high electron densities in the absence of static magnetic fields,[4] because its azimuthal field does not excite an ac space charge in an axially symmetric plasma. In the presence of a static magnetic field, this is no longer true. The nonisotropic nature of the plasma causes radial currents and radial fields, which contribute to the frequency shift. A first-order correction to the perturbation formula can be obtained by using a pseudo-static approximation, in order to compute the radial microwave field in the plasma. Plots of the frequency shift obtained are shown in Fig. 4, and they are compared with the plot obtained from the simple perturbation formula which neglects the radial fields. A striking feature is the oscillation in $\Delta f/f$. The magnitude of the resonance becomes larger as the ν_c becomes smaller.

The behavior of the Q value of the cavity also can be calculated as a function of the electron density. A calculation of this sort leads to the results shown in Fig. 5, where we see a resonance minimum in $\Delta(1/Q)$. In the vicinity of the resonance, the Q value of the cavity is so low that accurate measurements of $\Delta f/f$ are difficult to obtain. A quantitative experimental verification has not yet been made, although a resonance in the Q value of the cavity has been observed. Although we can predict qualitatively the behavior of $\Delta f/f$ and $\Delta(1/Q)$ with the electron density, we know that quantitatively our solutions are not correct. This is so because the assumptions about the field in the cavity, which were made in order to derive the basic equations, are not compatible with the nonisotropic nature of the plasma when the plasma radius is not negligible compared with the wavelength. It is well known that in a nonisotropic medium, a pure TE mode is not possible. This is also true for the region of the cavity outside the plasma. In the present case the field in the cavity and in the plasma is some superposition of the fields of the TE_{011} and the TM_{011} modes. An exact solution of this problem is possible. It results in a transcendental equation for the complex resonant frequencies of the cavity which is in the form of a 6×6 determinant, and it must be computed by numerical means.

TM_{0m0} (E Parallel to B)

Low Densities

When the microwave mode is such that the E field is parallel to the B field, the effective dielectric coefficient

[4] S. J. Buchsbaum and S. C. Brown, "Microwave measurements of high electron densities," *Phys. Rev.*, vol. 106, pp. 196–199; April 15, 1957.

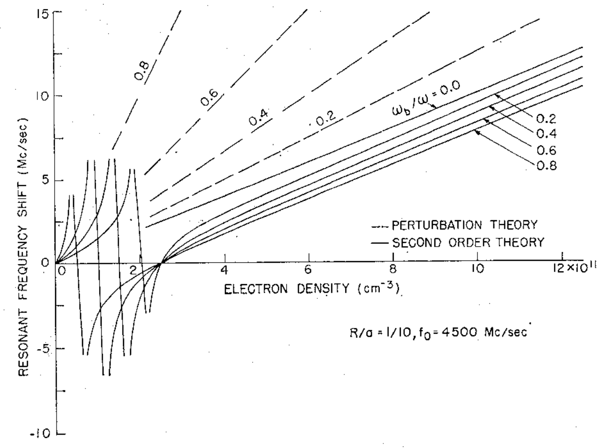

Fig. 4—Frequency shift as a function of electron density for the TE_{011} mode.

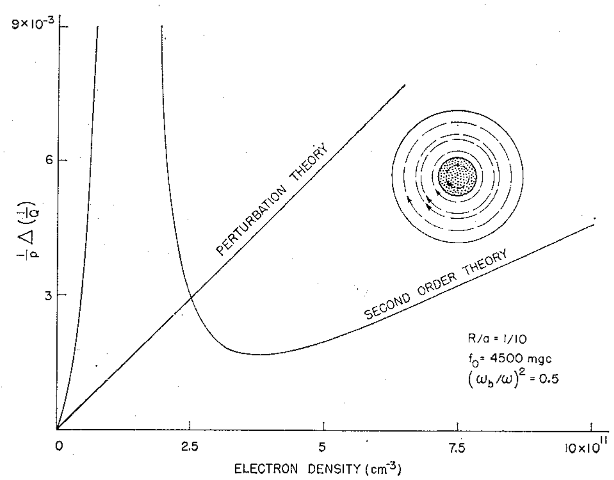

Fig. 5—The Q value as a function of electron density for the TE_{011} mode.

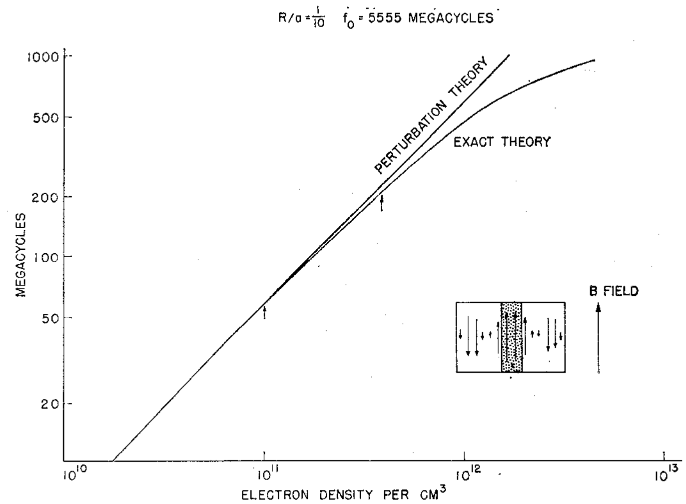

Fig. 6—Frequency shift as a function of electron density for the TM_{020} mode.

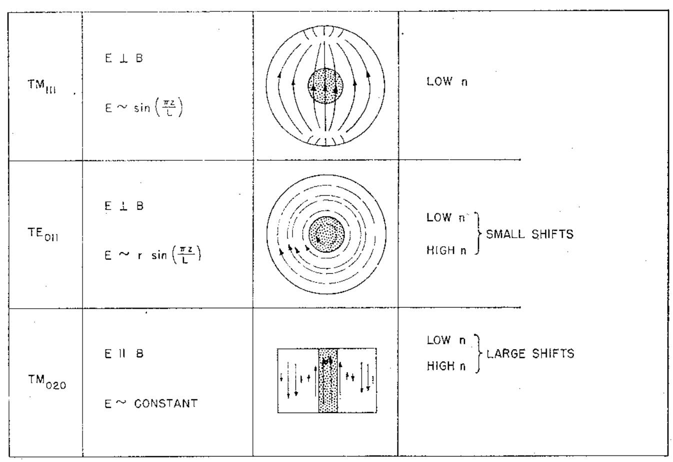

Fig. 7—Summary of field distributions for the TM_{111}, TE_{011}, TM_{020} modes.

is given by (2c). The frequency shift is independent of the magnetic field in the limit of zero electron temperature, and it is given by

$$\frac{\Delta f}{f} \sim \frac{\omega_p^2}{\omega^2}\left[\frac{1}{1+\frac{\nu_c^2}{\omega^2}}\right]. \qquad (5)$$

The solution of (5) for the frequency shift is given in Fig. 6; the curve is marked "Perturbation Theory."

High Electron Densities

Since the E field of these modes can be made to coincide with the direction of the static magnetic field, the TM_{0m0} modes do not suffer from the disadvantages that the nonisotropic nature of the plasma imposes on all other modes that have a component of the E field at right angles to the B field. Although the TM_{0m0} are not as ideal as the TE_{011} mode, they are well suited for measuring high electron densities in those plasmas that do not possess density gradients in the axial direction. The disadvantage of the TM_{0m0} modes lies in the fact that the shift in the resonant frequency of the cavity is large when the plasma density is high. Consequently, the perturbation formula is again inadequate and exact analysis must be resorted to. In this case, however, the analysis is fairly straightforward. Fig. 6 shows the frequency shift of a TM_{020} mode cavity as a function of plasma density, which is assumed to be uniform, whose radius is $\frac{1}{10}$ of the cavity radius.

Measurement of Electron Density Distribution

Since the E fields of the three modes that have been discussed have different radial and axial functional dependences, the simultaneous use of two of the three modes yields information about the electron density distribution along the appropriate directions. The use of the TM_{111} mode, with E approximately constant with R, and of the TE_{011} mode, with E varying as R, yields information about the density distribution along the radius. The use of the TM_{111} mode, with E varying as sin $(\pi z/L)$, and of the TM_{010} mode, with E approximately constant with z, gives the distribution along the axis of the plasma. These results are summarized in Fig. 7.